Truncated Euler Systems

Soogil Seo (Yonsei University)

It is known that the order of the class group Cl_K of real abelian field K is essentially equal to the order of the quotient E_K/C_K of the global units E_K by the circular units C_K of K. However, the structures of these two groups are usually very different. Motivated by the theory of circular distributions and the special units of Rubin, we introduce a filtration to E_K made from the so-called truncated Euler systems and conjecture that the associated graded module is isomorphic, as a Galois module, to the class group. We use Euler systems to give evidence for this conjecture.

Control Theorems for Abelian Varieties over Global Function Fields

Ki-Seng Tan (National Taiwan University)

We prove the function field analogue for Mazur's Control theorems for abelian varieties.

A geometric proof of Mordell's conjecture for function fields

Kezheng LI (Capital Normal University)

Let $\mathcal{C}, \mathcal{C}'$ be curves over a base scheme S with $g(\mathcal{C}) \geq 2$. Then the functor $T \mapsto \{$generically smooth T-morphisms $T \times_S \mathcal{C}' \to T \times_S \mathcal{C}\}$ from ((S-schemes)) to ((sets)) is represented by a quasi-finite unramified S-scheme. From this one can easily deduce that for any two integers $g \geq 2$ and g', there is an integer $M(g, g')$ such that for any two curves C, C' over any field k with $g(C) = g$, $g(C') = g'$, there are at most $M(g, g')$ separable k-morphisms $C' \to C$. This is a today's strong version of Mordell's conjecture for function fields. From the above method one can also see some interesting properties of higher dimensional varieties.

The non-existence of certain mod 2 Galois representations of some small quadratic fields

Yuichiro Taguchi (Kyushu University)

For some quadratic fields of small discriminant, we prove the non-existence of mod 2 Galois representations of degree 2 unramified outside 2 and the infinite place. This implies a few special cases of a version of Serre's modularity conjecture, of which the original version for the rational number field has been proved recently by Khare and Wintenberger.

(This is a joint work with Hyunsuk Moon.)

On Motivic Transcendence Theory in Positive Characteristic

Jing Yu (NCTS, National Tsinghua University)

I will give a survey of recent progress on t-motives in positive characteristic. In particular the development of a motivic Galois theory and its applications to algebraic independence problems. This leads to the determination of all algebraic relations of sets of transcendental invariants which arise naturally from function field arithmetic.

Bounded extensions of rank one (φ, Γ)-modules

Seunghwan Chang (KAIST)

I will talk about a parametrization of mod p extensions of rank one (φ, Γ)-modules and introduce the notion of bounded extensions, which (conjecturally) describe the distinguished subspaces in the recipe for weights in Buzzard-Diamond-Jarvis' generalization of (weight part of) Serre's conjecture over totally real fields.

Tower of the maximal abelian extensions of local fields and their application

Xu Fei (Chinese Academy of Sciences)

The local class field theory gives the explicit description of the maximal abelian extension of a local field and the corresponding Galois group. From representaion point of view, Langlands has formulated his famous local langlands program which extends the local class field theory to general non-abelian cases. This program has recently been proved by Harris and Taylor. In this talk we will study the tower of the successive maximal abelian extensions of local fields and give the explicit description of these fields and the Galois groups at each level. The main ingredient is to study the algebraic structue of the principal units in infinite abelian extensions of local fields by using Galois cohomology. As application, we show there are only finitely many finite covering of given degree of the maximal abelian extension of a local field. By using Φ-Γ-module, the number of finite covering of given degree of the maximal abelian extension of Q_p has been computed.

The transfer maps for motivic cohomology and Nesterenko-Suslin theorem

Sung Myung (Inha University)

Transfer Maps for Milnor's K-theory were first defined by Bass and Tate (1972) for simple extensions of fields via tame symbol and Weil's reciprocity law, but their functoriality had not been settled until Kato (1980). Meanwhile, certain motivic cohomology groups, defined in terms of higher Chow groups, are shown to be isomorphic to Milnor's K-groups of fields by Nesterenko and Suslin (1989). But, in fact, functorial transfer maps for motivic cohomology are easily defined using GW-complex. In this talk, we show that these natural motivic transfer maps actually agree with the classical but difficult transfer maps by Bass and Tate. A by-product of this result is another proof of Nesterenko and Suslin's theorem for this version of motivic complex.

$K_{2i}(O_F)$ for Z_p-extension

Hourong Qin (Nanjing University)

Let F be a number field with $\mu_p \subset F$ if p is odd and $\mu_4 \subset F$ if $p = 2$. Let F_∞/F be the cyclotomic Z_p-extension, and for all integers $n \geq 0$, let F_n be the unique intermediate field for F_∞/F such that $[F_n : F] = p^n$. Let O_F be the ring of integers in F and O_{F_n} be the integral closure of O_F in F_n. For $K_{2i}(O_{F_n})\{p\}$, the p-primary subgroup of $K_{2i}(O_{F_n})$, we establish an analogue of the classical Iwasawa theorem. We will discuss the structure of $K_{2i}(O_{F_n})\{p\}$ and study the relation between $K_{2i}(O_{F_n})\{p\}$ and the p-primary subgroup of the ideal class group of F_n. We will give some equivalent statements for the Vandiver conjecture to be true. This is the joint work with Qingzhong Ji.

On a Local-Global Property of Algebraic Dynamics

Liang-Chung Hsia (National Central University)

A key tool in the study of Diophantine equations is the application of local-global principles, such as the Hasse principle and the Brauer-Manin obstruction. In this talk, we'll discuss a question concerning local-global principle for algebraic dynamical systems. Let $\varphi : \mathbb{P}^N \to \mathbb{P}^N$ be a morphism of degree $d \geq 2$ defined over a number field K and let $P \in \mathbb{P}^N(K)$ be a point with infinite forward orbit $\mathcal{O}_\varphi(P)$. Let $\mathbf{A}_K$ denote the ring of adeles of K and write $\overline{\mathcal{O}_\varphi(P)}$ for the closure of $\mathcal{O}_\varphi(P)$ in $\mathbb{P}^N(\mathbf{A}_K)$. Let $V \subset \mathbb{P}^N$ be an irreducible variety. Then must it be true that either (i) $\mathcal{O}_\varphi(P) \cap V(K) = \overline{\mathcal{O}_\varphi(P)} \cap V(\mathbf{A}_K)$ or (ii) there are positive integers $n > m \geq 0$ such that $\varphi^n(V) = \varphi^m(V)$? We study this question for the d^{th} power map on $\mathbb{P}^2$ and give an affirmative answer for varieties V that are translates of tori in $\mathbb{G}_m^2$.

(This is a joint work with Joseph Silverman.)

Regular positive ternary quadratic forms

Byeong-Kweon Oh (Sejong University)

A positive definite quadratic form f is said to be regular if it globally represents all integers that are represented by the genus of f. In 1997, Jagy, Kaplansky and Schiemann provided a list of 913 candidates of primitive positive definite regular ternary quadratic forms, and all but 22 of them are verified to be regular. In this talk we prove that eight forms among 22 candidates are regular.

Congruences between extremal modular forms and theta series of special types modulo powers of 2 and 3

Masao Koike (Kyushu University)

Heninger, Rains and Sloane proved that the n-th root of the theta series of any extremal even unimodular lattice in $\mathbf{R}^n$ have integer coefficients if n is of the form $2^i 3^j 5^k (i \geq 3)$.Motivated by their discovery, we find the congruences between extremal modular forms and theta series of special types modulo powers of 2 and 3. This assertion enables us to prove that $2n$-th root and $3n$-th root of the extremal modular form of weight $\frac{n}{2}$ have at least a non-integer coefficient.

Vahlen's Involution and q-series identities

Joon Yop Lee (POSTECH)

After introducing Vahlen's involution which is a partition bijection, combinatorial proofs of some q-series identities will be given. Using this, generalizations will be considered. And derived identities will be explained in a similar context. Finally, identities in Ramanujan's lost notebook will be treated.

On the coefficients of certain family of modular equations

Nam Min Kim (KAIST)

The n-th modular equation for the elliptic modular function $j(z)$ has large coefficients even for small n, and those coefficients grow rapidly as $n \to \infty$. The growth of these coefficients was first obtained by Cohen. And, recently Cais and Conrad considered this problem for the Hauptmodul $j_5(z)$ of $\Gamma(5)$, and they found that the ratio of the growths of the coefficients of the modular equations for $j(z)$ and $j_5(z)$ is related to the group index $[\overline{\Gamma(1)} : \overline{\Gamma(5)}] = 60$. In this paper we extend this problem to Hauptmoduls of somewhat general Fuchsian groups of the first kind with genus zero.

On the Implementation of Tate Pairings

Soonhak Kwon (Sungkyunkwan University)

In this talk, we discuss recent progress on the computation of Tate pairing of elliptic curves. We explain various techniques of speeding up the computation of bilinear pairings such as Eta and Ate pairings.

An analogue of the discriminant function for congruence subgroup and its application to the extremal quasimodular forms

Hiroyuki Tsutsumi (Osaka University of Health and Sport Sciences)

We define and study an analogue of the discriminant function for congruence subgroup. The analogues for congruence subgroups of low levels are well known, but the ones corresponding to high levels are hardly known. Its application to the extremal quasimodular forms will be also discussed there.

This research is joint with Yuichi Sakai (Kyusyu University).

Prime Solutions to Quadratic Equations

Jianya Liu (Shandong University)

In this talk, I will try to explain how to find prime solutions to quadratic equations. Among other things, I will report a joint result with Peter Sarnak in this direction, obtained via the theory of quadratic forms, the Jacquet-Langlands theory on automorhic representations, the Kim-Sarnak bound toward the Selberg eigenvalue conjecture, and a three dimensional combinatorial sieve.

Extended Gauss AGM and isogenous formulas of modular forms

Hironori Shiga (Chiba University) joint work with Kenji Koike

In 1799 Gauss discovered the formula connecting the arithmetic geometric mean (we say AGM) and the Gauss hypergeometric function. Later he found the isogeny formula of Jacobi theta constants corresponding to the AGM process (before Jacobi).

These two formulas suggest a possibility of new field connecting arithmetc, algebraic geometry, hypergeometric functions and modular forms. Over 200 years this story has been almost forgotten.

May be now is the time to restart the study on AGM with wider aspect. As a first step we found two nice extensions of the AGM story connected with the Picard modular forms of two variables.

We shall explain the outline of these new extended AGMs.

Modular units and divisor class groups of the modular curves $X_1(N)$

Yifan Yang (National Chiao-Tung University)

In this talk, we consider the group $F(N)$ of modular units on $X_1(N)$ that have divisors supported on the cusps lying over ∞ of $X_0(N)$, called the ∞-cusps. For each positive integer N, we will give an explicit basis for the group $F(N)$. This enables us to compute the group structure of the rational torsion subgroup $C(N)$ of the Jacobian $J_1(N)$ of $X_1(N)$ generated by the differences of the ∞-cusps. Based on our numerical computation, we make a conjecture on the structure of the p-primary part of $C(p^n)$ for a regular prime p.

Calculation of l-adic local Fourier transformations.

Lei Fu (Nankai University)

The global l-adic Fourier transformation was first introduced by Deligne. To study the local behavior of the global Fourier transformation, Laumon discovered the stationary phase principle and introduced local Fourier transformations. All these transformations are defined by cohomological functors and are rarely computable. However, in Laumon's paper "Transformation de Fourier, Constantes d'Équations Fontionnelles, et Conjecture de Weil", Publ. Math. IHES 65 (1987), 131-210, Laumon and Malgrange give conjectural formulas of local Fourier transformations for a class of $\overline{\mathbf{Q}}_l$-sheaves. In this paper, we prove these conjectures. Actually we are able to calculate local Fourier transformations for a slightly more general class of $\overline{\mathbf{Q}}_l$-sheaves. It turns out that to get the correct result, the conjectural formulas of Laumon and Malgrange have to be slightly modified.

On the p-adic L-functions for modular forms

Masataka Chida (Tohoku University)

We will discuss about the cyclotomic and anti-cyclotomic p-adic L-functions associated to modular forms and relations to Selmer goroups. In particular, we consider the Iwasawa main conjecture for modular forms.

Ramakrishna-Khare Systems in Higher Weights

Yih-Jeng YU (National Center for Theoretical Science)

Let $\overline{\rho} : G_{\mathbb{Q}} \to \mathrm{GL}_2(\mathbb{F})$ be a continuous, absolutely irreducible mod p representation, with $\mathbb{F}$ a finite field of characteristic $p > 5$, and $G_{\mathbb{Q}}$ the absolute Galois group of the rational number field $\mathbb{Q}$. Assume $\overline{\rho}$ has Serre weight $k < p$. Under appropriate hypotheses on $\overline{\rho}$, we generalize a result of C. Khare stating that all its minimal deformations are modular. His result was based on a previous paper by Ramakrishna-Khare using "*auxiliary*" finite sets Q of prime numbers. Khare treated the case of weight 2; we treat the case of weights $2 \leq k < p$.

There are three main steps in the proof. The first is Ramakrishna-Khare theorem, valid for $k < p$. The second is to generalize results of Ribet and Ribet-Takahashi which were proven in weight 2. This is the core of this work to generalize these results to higher weights, following ideas of Jordan-Livné, Rajaei and Jarvis. The last is Böckle descent argument, also valid for $k < p$.

The first sign change of Fourier coefficients of cusp forms

YoungJu Choie (POSTECH)

Let f be a non-zero cusp form of even integral weight $k > 1$ on the Hecke congruence subgroup $\Gamma_0(N)$ with real Fourier coefficients $a(n)$. Then using a classical theorem of Landau on Dirichlet series with non-negative real coefficients coupled with the analytic properties of the Hecke L-function and Rankin-Selberg L-function attached to f, it is easy to see that $a(n)$ changes sign infinitely often. It is therefore quite natural to ask when the first sign change occurs and one would hope for an upper bound for it depending only on k and N. The above problem was studied by Kohnen and Sengupta in the special case where f is a normalized Hecke eigenform of squarefree level N that is a newform (so $a(1) = 1$ and $a(n)$ is the n-th Hecke eigenvalue using techniques from analytic number theory and properties of the symmetric square L-function $L(sym^2 f, s)$ attached to f.

In this paper we extend the method studied by Kohnen and Sengupta to arbitrary cusp forms f of squarefree level N.

This work is done jointly with W.Kohnen.