

# WORKSHOP

2009 Algebraic Geometry Workshop at KAIST

Daejeon, KOREA

일시: April 9(목) ~ 11(토), 2009

장소: 자연과학동 수리과학과 1409

후원: 과학재단

대수구조 및 응용연구센터

**Algebraic Structure and its Applications Research Center**

# Workshop Information

## □ Tentative Schedule

April 9 (Thu)	Speaker	April 10 (Fri)	Speaker	April 11 (Sat)	Speaker
09:30-10:00	Registration				
10:00-11:00	H.Lange	09:30-10:30	P.Vermeire	10:00-11:00	H.Park
11:20-12:20	I.Choe	10:50-11:50	H.Lange	11:20-12:20	P.Vermeire
Lunch		Lunch		Lunch	
14:20-15:20	C.Birkar	13:30-14:30	C.Miyazaki	13:30-	Excursion
15:40-16:40	J.Hwang	14:50-15:50	A.Noma		
		16:10-17:10	S.Huh		
		18:00-	Banquet		

## □ Speakers & Schedule :

### April 9 (Thursday)

9:30-10:00	<b>Registration</b>
10:00-11:00	<b>Herbert Lange (Universitat-Erlangen-Nurnberg)</b> <i>Clifford indices for vector bundles on curves</i>
11:00-11:20	<b>Coffee Break</b>

11:20-12:20	<b>Insong Choe (Konkuk University)</b> Segre invariants of vector bundles on curves and higher secant varieties
12:20-14:20	Lunch Time
14:20-15:20	<b>Caucher Birkar (University of Cambridge)</b> The Iitaka conjecture in dimension 6
15:20-15:40	<b>Coffee Break</b>
15:40-16:40	<b>Jun-muk Hwang (KIAS)</b> Characteristic foliation of a hypersurface of general type on a projective symplectic manifold

#### **April 10 (Friday)**

9:30-10:30	<b>Peter Vermeire (Central Michigan University)</b> Normality, equations and syzygies of secant varieties
10:30-10:50	<b>Coffee Break</b>
10:50-11:50	<b>Herbert Lange (Universitat-Erlangen-Nurnberg)</b> Clifford indices for vector bundles on curves
12:00-13:30	Lunch Time
13:30-14:30	<b>Chikashi Miyazaki (Saga University)</b> Regularity of projective varieties and classical Castelnuovo's methods
14:30-14:50	<b>Coffee Break</b>

14:50-15:50	<b>Atsushi Noma (Yokohama National University)</b> Projective varieties with nonbirational linear projection and applications
15:50-16:10	<b>Coffee Break</b>
16:10-17:10	<b>Sukmoon Huh (KIAS)</b> Geometry of moduli spaces of stable sheaves on surfaces and embedded curves
18:00-	<b>Banquet</b>

#### **April 11 (Saturday)**

10:00-11:00	<b>Hyungju Park (KIAS)</b> A generalization of Castelnuovo-Mumford regularity for representations
11:00-11:20	<b>Coffee Break</b>
11:20-12:20	<b>Peter Vermeire (Central Michigan University)</b> Normality, equations and syzygies of secant varieties
12:20-13:30	Lunch Time
13:30-	<b>Excursion</b>

## 1. Herbert Lange (Universitat-Erlangen-Nurnberg)

**Title :** [Clifford indices for vector bundles on curves I](#)

**Abstract:** This is a report on a recent joint paper with Peter Newstead. For smooth projective curves the Clifford index is an important invariant which provides a bound for the dimension of the space of sections of a line bundle. This is the first step in distinguishing curves of the same genus. In this paper we generalise this to introduce Clifford indices for semistable vector bundles on curves. We study these invariants, giving some basic properties and carrying out some computations for small ranks and for general and some special curves. For curves whose classical Clifford index is two, we compute all values of our new Clifford indices.

## 2. Insong Choe (Konkuk University)

**Title:** [Segre invariants of vector bundles on curves and higher secant varieties](#)

**Abstract:** In this talk, we discuss some generalizations of the old result of Lange and Narasimhan on the Segre invariant of rank 2 bundles over a curve. We consider the Segre invariants of the bundles of higher rank and reprove the Hirschowitz bound by using the secant varieties related to the ruled variety over the curve. We also study the Segre stratification on the moduli space of symplectic bundles of rank 4. This is a joint work with G. H. Hitching.

## 3. Caucher Birkar (University of Cambridge)

**Title:** [The Iitaka conjecture in dimension 6](#)

**Abstract:** I will discuss the Iitaka conjecture about the Kodaira dimension of algebraic fibre spaces, and describe a proof in the 6-dimensional case.

## 4. Jun-Muk Hwang (KIAS)

**Title:** [Characteristic foliation of a hypersurface of general type on a projective symplectic manifold](#)

**Abstract:** Given a projective symplectic manifold  $M$  and a non-singular hypersurface  $X$  in  $M$ , the symplectic form of  $M$  induces a foliation of rank 1 on  $X$ , called the characteristic foliation. We study the question when the characteristic foliation is algebraic, namely, all the leaves are algebraic curves.

Our main result is that the characteristic foliation of  $X$  is not algebraic if  $X$  is of general type. For the proof, we first establish an étale version of Reeb stability theorem in foliation theory and then combine it with the positivity of the direct image sheaves associated to families of curves. This is a joint work with E. Viehweg.

## 5. Peter Vermeire (Central Michigan University)

**Title:** [Normality, equations and syzygies of secant varieties, I](#)

**Abstract:** I will discuss the geometry of the variety of secant lines to a smooth projective variety. Using the work of Bertram and Thaddeus on the moduli of rank 2 vector bundles as a guide, I will show how one can quickly understand the singularities of secant varieties. I will then discuss joint work with Jessica Sidman regarding the regularity, normality, and Cohen-Macaulayness of the secant variety to a smooth curve.

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## 6. Herbert Lange (Universitat-Erlangen-Nurnberg)

**Title:** Clifford indices for vector bundles on curves II

**Abstract:** This is a report on a recent joint paper with Peter Newstead.

For smooth projective curves the Clifford index is an important invariant which provides a bound for the dimension of the

space of sections of a line bundle. This is the first step in distinguishing curves of the same genus. In this

paper we generalise this to introduce Clifford indices for semistable vector bundles on curves.

We study these invariants,

giving some basic properties and carrying out some computations for small ranks and for general and some special curves.

For curves whose classical Clifford index is two, we compute all values of our new Clifford indices.

## 7. Chikashi Miyazaki (Saga University)

**Title:** Regularity of projective varieties and classical Castelnuovo's methods

**Abstract:** I will talk on the next extremal case for a Castelnuovo-type bound  $\text{reg } C \leq |(\deg C - 1) / \text{codim } C| + \max\{k(C), 1\}$  for the Castelnuovo-Mumford regularity for a nondegenerate projective curve  $C$ , where  $k(C)$  is an invariant which measures the deficiency of the Hartshorne-Rao module of  $C$ . Also, in higher dimensional cases, for a nondegenerate projective Buchsbaum variety  $V$ , a bound  $\text{reg } V \leq \lceil (\deg V - 1) / \text{codim } V \rceil + 1$  gives the corresponding next extremal variety. The socle lemma by Huneke-Ulrich and a result from the Castelnuovo theory by Eisenbud-Harris plays an important role for the proof.

## 8. Atsushi Noma (Yokohama National University)

**Title:** Projective varieties with nonbirational linear projections and applications

**Abstract:** The purpose of this talk is to give a classification of projective varieties admitting linear projections from points off  $\mathbb{P}^n$  or of  $\mathbb{P}^n$  which induce nonbirational maps of  $\mathbb{P}^n$  onto their images. As applications, we will give some results about the defining equations of a projective variety.

## 9. Sukmoon Huh (KIAS)

**Title :** Geometry of moduli spaces of stable sheaves on surfaces and embedded curves

**Abstract :** The general philosophy is that moduli spaces of stable sheaves inherit a lot of properties of the underlying varieties, which are curves or surfaces in our cases. If a curve is embedded into a surface in a particular way, then we may expect that two moduli spaces are closely related. We see this phenomena in cases of non-hyperelliptic curves of genus 3 and 4 for which the surfaces would be the projective plane and a quadric surface, by studying the rational map between moduli spaces given by restriction.

## 10. Hyungju Park (KIAS)

**Title:** A generalization of Castelnuovo-Mumford regularity for representations

**Abstract:** We introduce and generalize the notion of Castelnuovo-Mumford regularity for representations of noncommutative algebras, effectively establishing a measure of complexity for such objects. A noncommutative analogue of Schreyer's Theorem is proved for computing syzygies. By a repeated application of this theorem, we construct free resolutions for representations of noncommutative algebras. Some interesting examples are included in which graded free resolutions and regularities are computed for representations of various algebras.

## 11. Peter Vermeire (Central Michigan University)

**Title:** Normality, equations and syzygies of secant varieties, II

**Abstract:** I will discuss the regularity, normality, and Cohen-Macaulayness of the secant variety to a smooth projective variety, extending work done together with Sidman for curves. I will then discuss how to adapt the Koszul techniques of Green and Lazarsfeld to understand the equations defining secant varieties as well as their syzygies.