

Syzygies in Algebra and Geometry 2015



August 26th - August 30th, 2015

Haeundae, Busan, KOREA



Schedule

August 26 (Wed)	Speaker	August 27 (Thu)	Speaker	August 28 (Fri)	Speaker	August 29 (Sat)	Speaker	August 30 (Sun)	Speaker
09:30-10:00	Registration								
10:00-11:00	Trung	09:30-10:30	Singh	09:30-10:30	Ahn	09:30-10:30	Dao	09:30-10:30	Discussion II
10:30-12:30	Park	11:00-12:00	Raicu	11:00-12:00	Kemeny	11:00-12:00	Shin	11:00-12:00	Discussion III
	Lunch		Lunch		Lunch		Lunch		
14:30-15:30	Boij	14:00-15:00	Aprodu	13:30-	Free afternoon	14:00-15:00	Conca		
16:00-17:00	Varbaro	15:30-16:30	Faenzi			15:30-16:30	Discussion I		
		17:00-18:00	Contributed talks						
		18:00-	Banquet						

Castelnuovo-Mumford regularity and Ratliff-Rush closure

N.V. Trung

Abstract

We establish relationships between the Castelnuovo-Mumford regularity of standard graded algebras and the Ratliff-Rush closure of ideals. These relationships can be used to compute the Ratliff-closure and the regularities of the Rees algebra and the fiber ring. As a consequence, these regularities are equal for large classes of monomial ideals in two variables, thereby confirming a conjecture of Eisenbud and Ulrich for these cases. This is a joint work with M.E. Rossi and D.T. Trung.

On syzygies of ruled surfaces

Euisung PARK

Abstract

In this talk, I will speak about the minimal free resolution of homogeneous coordinate rings of a ruled surface S over a curve of genus g with the numerical invariant $e < 0$ and a minimal section C_0 . Let $L \in \text{Pic}X$ be a line bundle in the numerical class of $aC_0 + bf$ such that $a \geq 1$ and $2b - ae = 4g - 1 + k$ for some $k \geq \max(2, -e)$. We prove that the Green-Lazarsfeld index $\text{index}(S, L)$ of (S, L) , i.e. the maximum p such that L satisfies condition $N_{2,p}$, satisfies the inequalities

$$\frac{k}{2} - g \leq \text{index}(S, L) \leq \frac{k}{2} - \frac{ae + 3}{2} + \max\left(0, \left\lceil \frac{2g - 3 + ae - k}{4} \right\rceil\right). \quad (1)$$

Also if S has an effective divisor $D \equiv 2C_0 + ef$, then we obtain another upper bound of $\text{index}(S, L)$, i.e., $\text{index}(S, L) \leq k + \max\left(0, \left\lceil \frac{2g - 4 - k}{2} \right\rceil\right)$. This gives a better bound in case b is small compared to a . Finally, I will show that for each $e \in \{-g, \dots, -1\}$ there exists a ruled surface S with the numerical invariant e and a minimal section C_0 which has an effective divisor $D \equiv 2C_0 + ef$.

Cones of Betti tables and Hilbert functions

Mats Boij

Abstract

Studying the possible Betti tables of graded modules, it turned out to be useful to relax the question and only look at the Betti tables up to scaling, i.e., to study the cone spanned by the Betti tables in a suitable vector space. In the standard graded case, Macaulay's theorem gives us a complete classification of Hilbert functions of cyclic modules, but in other cases we are lacking such a classification and results on cones of Hilbert functions are useful. We can also combine the two questions and look at Hilbert functions of modules with some properties seen from the Betti table but not directly on the Hilbert function, as in the case of modules with bounded regularity. I will give a survey on some of the work that has been done on cones of Betti tables and Hilbert functions over the last few years including some recent joint work with Gregory G. Smith.

Dual graphs of projective schemes

Matteo Varbaro

Abstract

Given a projective scheme X , its dual graph $G(X)$ is the graph whose vertices are given by the irreducible components of X , and such that 2 vertices are connected by an edge iff the intersection of the 2 correspondent components is a codimension 1 subscheme of X . A classical result of Hartshorne says that, if X admits an arithmetically Cohen-Macaulay (aCM) embedding, then $G(X)$ is connected. In a joint work with Bruno Benedetti and Barbara Bolognese, we improved the conclusion assuming that X admits an arithmetically Gorenstein embedding: in this case, if the (Castelnuovo-Mumford) regularity of X (in such an embedding) is $r + 1$, and the regularity of each irreducible component of X is $\leq d$, then $G(X)$ is $\lfloor (r + d - 1)/d \rfloor$ -connected. We also proved that for any graph there exists a reduced projective curve C admitting an aCM embedding in which all the irreducible components of C are rational normal scrolls. During the talk we will discuss these features, some examples, and open questions on dual graphs.

Cohomology of thickenings of projective varieties

Anurag K. Singh

Abstract

Let X be a smooth projective subvariety of \mathbb{P}^n over a field of characteristic zero. We discuss a version of the Kodaira vanishing theorem for thickenings of X in \mathbb{P}^n , and a related result on the injectivity of the natural maps from Ext modules to local cohomology modules. This is a joint work with Bhatt, Blickle, Lyubeznik, and Zhang.

The syzygies of some thickenings of determinantal varieties

Claudiu Raicu

Abstract

The space of $m \times n$ matrices admits a natural action of the group $GL_m \times GL_n$ via row and column operations on the matrix entries. The invariant closed subsets are the determinantal varieties defined by the \mathfrak{a}_r (reduced) ideals of minors of the generic $m \times n$ matrix. The minimal free resolutions for these ideals are well-understood by work of Lascoux and others. There are however many more invariant ideals which are non-reduced, and whose syzygies are quite mysterious. These ideals correspond to nilpotent structures on the determinantal varieties, and they have been completely classified by De Concini, Eisenbud and Procesi. In my talk I will recall the classical description of syzygies of determinantal varieties, and explain how this can be extended to a large collection of their thickenings. Joint work with Jerzy Weyman.

Cayley-Chow forms of $K3$ surfaces and Ulrich bundles

Marian Aprodu

Abstract

An Ulrich bundle on a projective variety is a vector bundle that admits a completely linear resolution as a sheaf on the projective space. Ulrich bundles are semi-stable and the restrictions to any hyperplane section remain semi-stable. This notion originates in classical algebraic geometry, being related to the problem of finding, whenever possible, linear determinantal or linear pfaffian descriptions of hypersurfaces in a complex projective space. Generally, the existence of an Ulrich bundle has nice consequences on the equations of the given variety, specifically, the Cayley-Chow form is the determinant of a matrix of linear forms in the Pluecker coordinates. We prove existence of stable rank-two Ulrich bundles on polarized $K3$ surfaces with a mild Brill-Noether condition. As a consequence, we obtain an explicit pfaffian representation of the associated Cayley-Chow form. This is a joint work with Gavril Farkas and Angela Ortega.

Betti Tables of ND(1)-schemes

Jeaman Ahn

Abstract

In this talk, we introduce ND(1)-schemes, which generalize the concept of ‘being nondegenerate’ from the case of varieties to the case of more general closed subschemes and give a geometric interpretation on Betti numbers of ND(1)-schemes. For this purpose, we use elimination mapping cone theorem and generic initial ideal theory. We also provide some illuminating examples of our results via calculations done with Macaulay 2.

Syzygies of curves via $K3$ surfaces

Michael Kemeny

Abstract

In this talk I will explain how polarised $K3$ surfaces can be used to study the syzygies of (general) embedded curves. In particular, such methods have recently been used to resolve two conjectures which can be considered as analogues of Green’s conjecture on the minimal free resolution of the ideal sheaf of a canonically embedded curve. The first result is a proof of the Prym-Green conjecture for Prym-canonically embedded curves of odd genus; i.e. the Prym-canonical embedding of a general such curve has a natural resolution. The second result is a proof of the generic case of a famous conjecture of M. Green and R. Lazarsfeld relating property N_p to secants of embedded curves. If time permits I will also discuss recent progress on the higher level analogue of the Prym-Green conjecture. All results are joint with G. Farkas.

On some recent consequences of Serre's condition S_l

Hailong Dao

Abstract

Serre's condition S_2 is quite familiar to commutative algebraists as part of the condition for normality. On the surface it appears to be a rather weak condition. In this talk we will discuss some surprising consequences of S_l for $l = 2$ and higher. These consequences involve cohomological dimension, depth, h -vector, and in the square-free monomial case, the Castelnuovo-Mumford regularity of relevant ideals. We will also discuss some intriguing open questions on the tightness of these statements. This talk is based on recent joint work and discussions with D. Eisenbud, K. Han, S. Takagi, and M. Varbaro.

The minimal graded free resolution of a star-configuration in \mathbb{P}^n and secant varieties

Yong-Su Shin

Abstract

For positive integers r and s with $1 \leq r \leq \min\{n, s\}$, suppose F_1, \dots, F_s are general forms in $R = \mathbb{K}[x_0, x_1, \dots, x_n]$ of degrees d_1, \dots, d_s , respectively. We call the variety X defined by the ideal $\cap_{1 \leq i_1 < \dots < i_r \leq s} (F_{i_1}, \dots, F_{i_r})$ a star-configuration in \mathbb{P}^n of type (r, s) . In particular, if F_1, \dots, F_s are general linear forms in R , then we call X a linear star-configuration in \mathbb{P}^n of type (r, s) . As an application, we introduce secant varieties to the varieties of reducible forms.

Koszul algebras and their homological properties

Aldo Conca

Abstract

Koszul algebras are certain algebras defined by quadratic relations. Not all quadratic algebras are Koszul but most of the quadratic algebras that arise naturally are in fact Koszul. Unfortunately there is no finite criterion, no algorithm, to test Koszulness. In my talk I will explain some special features of syzygies of and over Koszul algebras. The talk is based on two recent papers:

Avramov, Luchezar L.; Conca, Aldo; Iyengar, Srikanth B. Subadditivity of syzygies of Koszul algebras *Math. Ann.* 361 (2015), no. 1-2, 511–534. arXiv:1308.6811

and

Conca, Aldo, Iyengar, Srikanth B., Nguyen Hop, Romer, Tim, Absolutely Koszul algebras and the Backelin-Roos property, *Acta Mathematica Vietnamica* 2015 arXiv:1411.7938

On the Koszul property for numerical semigroup rings

Dumitru Stamate (University of Bucharest & IMAR)

Abstract

Let H be a numerical semigroup. We give effective bounds for its multiplicity $e(H)$ such that $gr(K[H])$ is Koszul. We conjecture that not all the values in the range are possible, and this correlates to a series of conjectures of Eisenbud, Green and Harris on a Generalized Cayley-Bacharach statement. We describe the Koszul property for several classes of numerical semigroups and we study the relationship with the Cohen-Macaulay property of the $gr(K[H])$.

Joint work in progress with Juergen Herzog.

Varieties of powers and the Fröberg conjecture

Alessandro Oneto (Stockholm University)

Abstract

Motivated by a Waring problem for higher degree forms, i.e. on additive decomposition of degree kd homogeneous polynomials as sum of k -th powers, we study the secant varieties of varieties parametrizing k -th powers of degree d forms. This is related to the Fröberg conjecture on the Hilbert series of generic ideals. In this talk, we want to describe the relation between the two problems and the current situation about these questions. It is mainly based on joint works with R. Fröberg, G. Ottaviani and B. Shapiro.

Toric rings associated with isotone maps between posets

Ayesha Asloob Qureshi (Osaka University)

Abstract

Our main goal is to study the toric ring $K[P, Q]$ which is generated by the monomials arising from isotone maps from P to Q . Such rings are a natural generalization of the classical Hibi rings. We investigate the conditions on P and Q to determine when defining ideal of $K[P, Q]$ is quadratically generated and when it has square-free generators. Also, we compute the dimension of $K[P, Q]$.