A CONVEXITY THEOREM FOR THREE TANGLED HAMILTONIAN TORUS ACTIONS

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Abstract The convexity theorem for Hamiltonian torus actions states that the image of a moment map of a Hamiltonian torus action on a compact, connected symplectic manifold is a convex polytope ([A], [G-S]). Kirwan generalized this theorem to the case of any compact, connected Lie group, which also gives us a convex polytope ([K]). On the other hand, if a torus which has half the dimension of the manifold acts effectively in a Hamiltonian fashion, then its moment map provides completely integrable systems. Many important completely integrable systems are super-integrable systems in which each trajectory is contained in a smaller torus than the Liouville torus: harmonic oscillators, the Kepler system, the Toda lattice, etc. In this talk, motivated by the structure of the super-integrable system on the Toda lattice ([A-D-S]), a new generalization of the convexity theorem for Hamiltonian torus actions will be demonstrated.

References

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