An Erdős–Ko–Rado Theorem for cross t-intersecting families

Sang June Lee
KAIST
Algebraic Structure and its Applications Research Center
sjlee242@gmail.com

A central result in extremal set theory is the *Erdős–Ko–Rado Theorem* (1961) which investigates the maximum size of families $\mathcal{A} \subset \binom{[n]}{k} := \{S \subset [n] : |S| = k\}$ such that for every choice of $A_1, A_2 \in \mathcal{A}$ we have $|A_1 \cap A_2| \geq t$.

Two families $\mathcal{A}, \mathcal{B} \subset \binom{[n]}{k} := \{S \subset [n] : |S| = k\}$ are *cross t-intersecting* if for every choice of subsets $A \in \mathcal{A}$ and $B \in \mathcal{B}$ we have $|A \cap B| \geq t$. The following was conjectured as the cross t-intersecting version of the Erdős–Ko–Rado Theorem: For all $t \geq 1$, $k \geq t$ and $n \geq (t+1)(k-t+1)$, the maximum value of $|\mathcal{A}||\mathcal{B}|$ for two cross t-intersecting families $\mathcal{A}, \mathcal{B} \subset \binom{[n]}{k}$ is $\binom{n-t}{k-t}^2$.

In this talk we verify this for $t \geq 14$, large enough k (depending on t and any $\delta > 0$), and $n \geq (t+1+\delta)k$. Note that this range of n is arbitrarily close to $n \geq (t+1)(k-t+1)$ in the conjecture if δ is small and k is large. Our proofs make use of a *weight* version of the problem and *randomness*.

This is joint work with Peter Frankl, Norihide Tokushige, and Mark Siggers.