MOMENT-ANGLE MANIFOLDS IN TORIC TOPOLOGY

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1. From real quadrics to polytopes via manifolds.

Manifolds obtained as complete intersections of real quadratic hypersurfaces in a complex space have a natural torus action on them, and are known to toric topologists as moment-angle manifolds. They correspond naturally to combinatorial simple polytopes, and the direct passage from quadrics to polytopes involves some nice convex geometrical reasoning. The quadratic equations or the polytopes may be very simple, while the corresponding moment-angle manifolds usually are quite complicated topologically. Despite the defining equations are real, moment-angle manifolds are complex. Their complex structures are neither Kähler nor symplectic, so moment-angle manifolds generalise the known non-Kähler series of Hopf and Calabi-Eckmann. Studying their topology proves to be an interesting and challenging problem.

2. Moment-angle complexes and coordinate subspace arrangements.

Here we shall discuss moment-angle complexes Z_K corresponding to simplicial complexes K. There is a deformation retraction from the complement $U(K)$ of the coordinate subspace arrangement corresponding to K onto Z_K. We review the cohomology rings of moment-angle complexes Z_K and complements $U(K)$ and then discuss what is known about their homotopy or topological types. The homotopy type of $U(K)$ for some complexes K may be understood by using unstable homotopy-theoretic methods, while the topology of moment-angle manifolds corresponding to certain polytopes can be described using equivariant surgery techniques. These two approaches have different power, although the results obtained by the two are remarkably similar. The nature of this similarity is yet to be understood.

3. Moment-angle complexes from simplicial posets.

The construction of moment-angle complexes may be extended from simplicial complexes to simplicial posets. As a result, a certain T^m-space Z_S is associated to an arbitrary simplicial poset S on m vertices. Face rings $Z[S]$ of simplicial posets generalise those of simplicial complexes, but have much more complicated algebraic structure. These rings $Z[S]$ may be studied by topological methods. The space Z_S has many important topological properties of the original moment-angle complex Z_K associated to a simplicial complex K. In particular, the integral cohomology algebra of Z_S is isomorphic to the Tor-algebra of the face ring $Z[S]$. This leads directly to a generalisation of Hochster's theorem, expressing the algebraic Betti numbers of the ring $Z[S]$ in terms of the homology of full subposets in S. Finally, the total amount of homology of Z_S may be estimated from below, which settles Halperin's toral rank conjecture for the moment-angle complexes Z_S.