Title: ABC Theorems and Büchi's Problem over Function Fields

Abstract: Hilbert's Tenth Problem asks whether there is a general algorithm to determine, given any polynomial in several variables, whether there exists a zero with all coordinates in $\mathbf{Z}$. It was proved in the negative by Yu. Matiyasevich in 1970. In the 70's J. R. Büchi attempted to prove a similar statement for a system of quadric equations, and he was able to relate it to the following Diophantine problem:

Conjecture (Büchi's square problem). *There exists an integer $M > 0$ such that all $x_1, ..., x_M \in \mathbf{Z}$ satisfying the equations*

$$x_1^2 - 2x_2^2 + x_3^2 = x_2^2 - 2x_3^2 + x_4^2 = \cdots = x_{M-2}^2 - 2x_{M-1}^2 + x_M^2 = 2$$

must also satisfy $x_i^2 = (x+i)^2$ for a fixed integer x and $i \in \{1, ..., M\}$.

A generalization of Büchi's square problem asks is there a positive integer integer M such that any sequence $(x_1^n, ..., x_M^n)$ of n-th powers of integers with n-th difference equal to $n!$ is necessarily a sequence of n-th powers of successive integers.

In this talk, we will first introduce some second main theorems over function fields and then use them to study generalized Büchi's problem for algebraic functions. The method also works for meromorphic functions and non-archimedean meromorphic functions.